ABSTRACT

The dynamic behaviour of ferromagnetic particle suspensions is of interest in various technical applications, including separation processes in the mining industry. In this work, an implementation of magnetic particle interactions within a Stokesian dynamics simulation framework is presented.

Stokesian dynamics is a particle-based, mesh-free method developed for nm to mm size particles in static or flowing liquid suspensions. Particle-fluid and pairwise hydrodynamic interactions are included in the technique, and additional external or interparticle forces can be included depending on the size of particles and the properties of the system under consideration. In this work, we include models to allow for 3D simulation of magnetic particles in the presence of an external magnetic field. A single magnetic point source can be arbitrarily assigned within the simulation domain. The magnetic interaction model includes macroscopic particle-field interactions as well as pairwise interactions between nearby particles, resulting from field-induced particle magnetization.

Results are compared with laboratory tests using a bench-scale flow cell. It is demonstrated that Stokesian dynamics is a feasible technique for numerically studying the behaviour of ferromagnetic systems. The applicability of the method for understanding and predicting particle level mechanisms of relevance for the efficiency and selectivity of mineral separation processes such as wet low-intensity magnetic separation (LIMS) is discussed.

1 Introduction

The study of ferromagnetic particle suspensions is relevant for better understanding of wet-state separation processes in mineral processing and recycling (Stener et al. 2014; Menad et al. 2014; Svoboda, Fujita 2003) as well as various other applications or processes based on magnetic properties of particles, for instance magnetorheology, micro-robotics and medical applications (Costa, Costa Banco 2009; Lu et al. 2014; Rosengart et al. 2005; Lunov et al. 2011).

Numerical simulation of ferromagnetic particles in suspension have been conducted within a number of probabilistic and deterministic particle dynamics simulation disciplines, including Monte-Carlo simulation (Satoh et al. 1996; Satoh 2008), LB-DEM (Han et al. 2010a,b) and Stokesian dynamics (Satoh et al. 1998; Satoh 2002; Irisa, Yokomine 2007). The focus of these studies have not been directed towards mineral separation processes, but rather novel uses of magnetic effects in applications related for instance to nanotechnology, ferrofluids and drug delivery systems. One of the more accurate methods related to evaluating the combined effect of liquid phase flow and magnetic effects is the LB-LES-DEM approach (Lattice-Boltzmann-Large Eddy Simulation-Discrete Element Method). Work in this direction has been described by
Han et al. (2007; 2010a; 2010b), but the method is restricted to a very limited number of particles due to the high computational expense.

There are some relatively recent studies utilising combined DEM-FEM-CFD simulation approaches for studying high-gradient magnetic separation (HGMS) and wet low-intensity magnetic separation (LIMS). Such work has been published by Lindner et al. (2013) and Murairu (2013). These types of coupled methods are beneficial in offering one or sometimes two-way coupling between continuum and discrete simulation disciplines by using commercially available simulation packages. The drawbacks mainly come from inherent limitations of each individual method or the coupling between methods. This can for instance be exemplified by the typical one-way hydrodynamics coupling or otherwise simplified description of the full interparticle hydrodynamics, limited possibilities to describe free surface interactions, etc. Although enabling the simulation of fairly large systems and allowing a relatively accurate representation of the macroscopic magnetic field, these methods are not able to account for complex mechanisms that can arise as result of combined effects of magnetic, hydrodynamic and possibly other relevant particle-level interactions.

The purpose of this work is to establish an intermediate simulation framework, which can combine the accuracy of suspension-oriented particle methods with the efficiency of a mesh-free mathematical description of the magnetic field and fluid flow. The model is illustrated and qualitatively evaluated in the context of a number of simulation examples. We also discuss wider applicability to the model for numerical simulation-based evaluation of magnetic separation efficiency.

2 Method

2.1 Stokesian dynamics

The Stokesian dynamics technique was originally developed in the 1980’s for the study of many-body interactions in non-equilibrium suspensions (Bossis, Brady 1984; Brady, Bossis 1988; Brady 2001). At that time, the main focus of simulations was directed towards studying the behaviour of hydrodynamically interacting particles in suspensions under various flow conditions. Due to the possibility to include additional interaction models, Stokesian dynamics implementations have since then been developed that for instance enables consideration of colloidal interactions, Brownian motion and interparticle effects from added dispersant (Toivakka et al. 1997; Nopola et al. 2004; Sand et al. 2009). There has also been much work devoted to increasing the computational efficiency of the technique (Bousfield 1990; Sierou, Brady 2001).

Earlier approaches using the Stokesian dynamics technique for studying ferromagnetic particle suspensions have included the definition of the external magnetic field as a homogeneous field covering the full simulation domain, as well as describing particles as being identical and of monodisperse size distribution (Satoh et al. 1998; Irisa, Yokomine 2007). Efforts have also been made to reduce the computational cost by introduction of a cluster-based approach (Satoh 2002).

Stokesian dynamics simulations are based on the N-body coupled Langevin equation (Brady, Bossis 1988), which is a variant of Newton’s second law of motion. It can be written in the form

\[ \mathbf{m} \frac{d\mathbf{u}(t)}{dt} = \mathbf{F}^H + \mathbf{F}^P + \mathbf{F}^M, \]  

(1)
where mass, \( m \), times the change in velocity, \( \frac{d\mathbf{v}(t)}{dt} \), is a function of the various forces influencing the particles. These forces are commonly divided into hydrodynamic forces, \( \mathbf{F}^H \), interparticle forces, \( \mathbf{F}^P \), and single particle forces such as for instance resulting from macroscopic forces of varying origins which influence particles, \( \mathbf{F}^M \). Due to the additivity of various forces into the equation describing the motion of particles in a suspension, interactions beyond the typical hydrodynamics and particle interaction models can be included in a relatively straightforward manner. For reducing the computational expense, it is often necessary to conduct an analysis of governing forces through the calculation of non-dimensional numbers. This analysis includes a relative comparison between different forces in order to determine their relative significance. In Stokesian dynamics, the most important non-dimensional number is the particle Reynold’s number, \( Re_p \). It describes the relative importance of inertial versus viscous effects as

\[
Re_p = \frac{\rho \mathbf{a}_{\text{char}} u_{\text{char}}}{\mu},
\]

where \( \rho \) and \( \mu \) are the density and viscosity of the liquid medium, respectively, \( \mathbf{a}_{\text{char}} \) the characteristic particle size and \( u_{\text{char}} \) the velocity of particles relative to the liquid. If the particle Reynolds number is small, defined as \( Re_p \ll 1 \), it can be assumed that viscous effects dominate and particles can be expected to react instantaneously to forces influencing them. This approximation is relevant for small particles that are freely moving with the fluid, and thus suitable for suspension flow simulations.

While this form of numerical analysis is relatively straightforward in steady-state-type systems, it becomes complicated in systems with rapidly changing conditions or where conditions vary considerably at different locations of the domain.

A benefit of the Stokesian dynamics technique is that it is developed for particles in suspensions and thus allows for study of high-concentration suspension behaviour without need for the computationally more costly DEM-CFD or DEM-FEM coupling or less accurate tracer particle simulations by using continuum-type approaches. The Stokesian dynamics technique allows relatively straightforward addition of force models describing various types of particle interactions in liquid media. Thus, colloidal interaction models, steric repulsion models from added dispersant and Brownian motion has previously been implemented into the simulation framework. In addition, the Stokesian dynamics approach utilised in this work includes CFD-adaptations to the hydrodynamics model expressions in order to improve the accuracy of simulations when studying particle systems of wider particle size distributions than the 1/10 size ratio traditionally considered as a limitation in Stokesian dynamics (Kim, Karrila 1991). This adaptation is described in detail by Sand et al. (2009) and Nopola (2004).

### 2.2 Magnetic interactions

An aim of this work was to create a flexible implementation for simulation of ferromagnetic particle suspensions under influence of three-dimensional magnetic fields with an arbitrarily assignable magnetic source. This is advancing the previous state-of-the-art in Stokesian dynamics simulations, where suspension behaviours have been studied in uniform, homogeneous, one-dimensional external magnetic fields (Satoh et al. 1996; Satoh 2002; Irisa, Yokomine 2007).
Particle motion and interactions related to magnetic effects are in this work based on three force models. This includes interaction between the particles and a stationary magnetic point source, an interparticle magnetisation model and a magnetic particle cluster orientation model. There are a number of assumptions made in order to increase the computational efficiency:

- Particle magnetisation is assumed to be homogeneous, and vary depending on the local magnetic field, which is a function of the distance to the magnetic point source. The magnetic point source is furthermore viewed as an infinitely long dipole.
- Particles are assumed to always be oriented in the direction of the magnetic field, as defined by the position of the magnetic point source. Consequently, particles are not specifically described as magnetic dipoles. Their magnetic dipole orientation is therefore always known and calculation of magnetic torque on individual particles can be omitted.
- Particles which are defined as being in close contact with each other in the vicinity of the magnetic pole are considered as being part of a magnetic cluster, whereby an additional force model works to orientate the particle cluster with the magnetic field. This can be considered as a cluster-based torque model, which applies for particle pairs but not for individual particles.

The relationship between magnetic susceptibility, $S$, the intensity of magnetisation, $M$, and the applied magnetic field, $H$, for various materials is discussed by Wills and Napier-Munn (2006). The susceptibility of paramagnetic and diamagnetic materials is nearly constant, while the properties of ferromagnetic materials are more complex. Modelling of magnetisation curves has previously been attempted (Abuelmaatti 1993), but due to the relative complexity of these expressions a simplified description was estimated for use in this work. In order to produce a realistic magnetisation behaviour of ferromagnetic particles, it was therefore assumed that the intensity of magnetisation, $M$, is a function of the saturation magnetisation, $M_s$, a magnetisation gradient, $g$, and magnetic field $H$ according to the expression

$$ M = M_s |e^{-gH} - 1|. $$  \hspace{1cm} (3)

Particle magnetisation based on their distance from the magnetic point source is assumed to follow this expression. As can be seen, this does not include any memory function for the magnetisation of particles.

![Figure 1. Magnetisation curve of ferromagnetic material (magnetite) with saturation magnetisation 0.38 T based on Eq. (3), as compared with data provided in Wills and Napier-Munn (2006).](image-url)
The magnetic dipole-dipole interaction force between two nearby particles can be defined as

$$\vec{F}_{ab} = \frac{3\mu_0}{4\pi|r|^3} \left[ (\vec{r} \times \vec{m}_a) \times \vec{m}_b + (\vec{r} \times \vec{m}_b) \times \vec{m}_a - 2\vec{r}(\vec{m}_a \cdot \vec{m}_b) + 5\vec{r}((\vec{r} \times \vec{m}_a) \cdot (\vec{r} \times \vec{m}_b)) \right]$$

(4)

Where $|\vec{r}|$ is the distance between the two magnetic dipole moments, $\vec{m}_a$ and $\vec{m}_b$, $\vec{r}$ the unit vector pointing from $\vec{m}_a$ to $\vec{m}_b$ and $\mu_0$ is the magnetic constant (Yung et al. 1998). The algorithmic implementation of the model is simplified due to the assumption that the particles are always oriented in direction of the external magnetic field. The relative (non-dimensional) force magnitude and direction between two magnetically interacting particles in a homogeneous magnetic field is for illustration shown in Figure 2.

![Figure 2. Direction (left) and relative force magnitude (right) of interaction between two magnetised particles.](image)

### 2.3 Algorithmic techniques

Various algorithmic techniques are typically incorporated in Stokesian dynamics codes in order to improve the computational efficiency of simulations. The implementations in this work include a sparse matrix solver as well as a neighbour list technique which allows the calculation of only pairwise interaction of nearby particles. This significantly reduces the computational efforts (Toivakka 1997; Sand et al. 2009). However, long-range interactions originating from the macroscopic magnetic field is not compatible with the neighbour list approach. In the algorithmic implementation, the attraction of particles to the magnetic source is instead considered as a single particle force. Interparticle forces resulting from magnetisation and particle cluster orientations are still considered within the neighbour list. This admits the consideration of long-range magnetic attraction forces simultaneously as the numerical implementation for particle-level interactions remains relatively efficient.
3 Results

3.1 Laboratory-scale flow cell

The setup of simulation examples presented in this work was constructed to allow comparison and later possibly validation with a laboratory scale flow cell, which can be equipped with an ultrasound transducer and magnets. The dimensions of the flow cell are 50x75mm (depth and width, respectively) and the length is 1000mm. 50mm is a typical distance between the magnetic drum and the tank bottom in wet LIMS, while 75mm is wide enough to avoid unwanted interaction between the ultrasound signal and the side walls. The ultrasound transducer enables the tracking of flow profile and localised solids concentrations in the suspension. If equipped with magnets, magnetic material can be captured as shown in Figure 3. This allows the monitoring of magnetic material build-up near the magnet, where factors to study might for instance include rate of build-up and cake thickness as function of flow rate. Experimentally tested suspensions within this flow cell have included magnetite particles, $d_{50}=34\mu$m, and at a concentration of 5-10 vol% at a suspension flow rate of up to 1 m/s (Stener et al. 2014). The experimental setup shown in Figure 3 involves a magnetite suspension flowing in the channel and magnetic material forming a cake at the wall near the magnet. The ultrasound transducer is omitted in the figure.

![Figure 3. Flow channel with magnet (top right) and magnetite particles aggregated to the channel wall (cake) below the magnet, as seen in the upper right section of the channel.](image)

3.2 Simulation setup

The simulation examples given below are designed to roughly correspond to the setup of the flow cell. A pressure-driven (parabolic) flow profile is used to describe the flow of the suspension. Solids concentrations up to 13 vol-% have been successfully simulated, which is comparable to the slurry concentrations used in the flow cell and also similar to the conditions in a magnetic separator. Due to the large number of particles (billions per cm$^3$) in the flow cell, its full representation within a simulation domain is not possible. Instead, the simulations below should be considered as representing only a small volume of the cell, typically less than 0.1
mm$^3$. Only monodisperse systems of spherical particles are considered in simulations, with particle sizes of 2-5 µm. The Stokesian dynamics approach presented in this work can, however, also be utilized for simulation of particle size distributions.

3.3 Simulation examples

The above-described simulation approach is evaluated in a number of test cases to demonstrate the functionality of the constituent models and how they interact. These examples should not be considered as a validation, but rather a demonstration that reasonable results can be obtained. Given the considerations above, the examples demonstrated in this work should thus be considered as giving indications of model capabilities rather than quantitative information on mechanisms or events taking place in the flow cell or in real magnetic separators.

Particle tracking can aid in the understanding of capture criteria and near-magnet flow patterns based on the combined effect of hydrodynamics, magnetic and other relevant forces. An example of such a simulation is shown in Figure 4. Particles have at the start of the simulation been randomly placed in the simulation domain and a parabolic flow profile applied. It can be seen that particles close to the defined magnetic point source are able to migrate against the flow direction towards the magnet. If the distance to the magnet source is too long, particles attracted to it might not be able to reach it before being carried away with the flow. This is especially noted if particles need to pass the high flow velocity region in the centre of the flow channel.

![Figure 4. Trajectories of magnetic particles near magnetic pole.](image)

It is known that non-magnetic particles in some cases can be captured within magnetic particle clusters and consequently carried with the concentrate. The mechanisms of such non-magnetic particle entrainment can be studied by numerical simulation, for instance by investigating the interrelation between macroscopic suspension flow, interparticle hydrodynamic effects and magnetic forces. Stokesian dynamics is a highly suitable technique in this respect as it includes full representation of hydrodynamic particle interactions through the liquid medium, while typical CFD-DEM or FEM-DEM approaches are based on a more simple approach where particles move along the liquid flow lines (Murariu 2013), but do not interact hydrodynamically with each other. This is especially relevant in high-concentration suspensions, where the calculation of hydrodynamic interactions in Stokesian dynamics strongly contributes to increased simulation accuracy (Nopola 2004). It also needs to be noted, however, that neither
this approach nor the CFD-DEM or FEM-DEM-work discussed above include two-way coupling between the liquid phase and particles.

A test simulation was carried out with a suspension of 1000 magnetic and an equal number of non-magnetic particles, Figure 5. Two distinct mechanisms that could result in particle entrapment were observed. The first being non-magnetic particles getting dragged or pushed by hydrodynamic or mechanical interactions with magnetic particles translating towards the magnetic source. The second mechanism was the flow of non-magnetic particles towards the formed magnetic cake by simple hydrodynamic drag by the flowing liquid medium. When suspended magnetic particles approach from behind and lock into the magnetic particle cluster, they may completely surround the non-magnetic particle before it has the chance to escape.

In addition to particle entrapment, the simulation also shows the formation of strings of magnetic particles, which orientate in the direction of the flow. This is a result of particle magnetisation and consequently attraction of particles to each other in the vicinity of the magnetic pole. Strings of magnetic particles can also eventually detach from the structure as seen in subfigures E and F.

As the number of particles in simulations is relatively low, an alternative type of visualisation was tested. Instead of visualising individual particles, sticks were used to describe particle contacts. The position of sticks can then in turn be viewed as a high concentration region of particles or as an indicator of particle aggregation due to magnetic or other forces. Such an example is shown in Figure 6. This figure can be compared with the visually observable magnetic cake in the experiment using the flow cell in Figure 3. Results appear very similar in terms of magnetic cake structure, with a strong build-up near the magnet and a tail in the

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**Figure 5.** Motion of 2µm magnetic particles (light) and non-magnetic particle (dark) entrapment in the particle cluster near the magnetic pole.
direction of the flow. It needs to be pointed out that since only particle clusters are illustrated in this example, individual particles that are not considered as forming part of a cluster are not at all shown in this mode of visualisation.

Figure 6. Visualisation of particle contacts as sticks. Magnetic material flowing through the cell captured by the magnet (A), cake buildup (B and C) and a steady state-type condition reached (D).

The three examples discussed above illustrate some visualisation possibilities and results that can be obtained using the method developed in this work. The simulation outcomes are logical in comparison with previous simulation work, experimental results and theory discussed in the introduction and method. Consequently, Stokesian dynamics appears to be a suitable method for simulation of magnetic phenomena in suspensions. With further development it is deemed to allow for relatively comprehensive studies to be conducted, related to mechanisms and interactions in magnetic separators. This could for instance include the effects of changes in material throughput and magnetic field strength on the selectivity of magnetic separation processes.

4 Concluding remarks

In this work, we have adapted a model for magnetic interactions to a Stokesian Dynamics simulation framework. This gives the capability to study the three-dimensional behaviour of ferromagnetic particle suspensions of varying particle size distributions subjected to a magnetic field.

This work addresses some of the issues of previously conducted modelling and simulation work related to magnetic interactions in suspensions. These include lacking or having simplified descriptions of particle hydrodynamic interactions, limitation to monodisperse systems or magnetic field representation. These issues may seriously restrict the predictive value of earlier methods, especially with relation to particle interaction mechanisms resulting from the combination of magnetic and hydrodynamic forces. This strongly motivates the use of the
Stokesian dynamics technique and this modified approach in particular, which allows more accurate hydrodynamic representation for polydisperse particle size distributions (Sand et al. 2011).

The model allows systematic study of various mechanisms or properties related to ferromagnetic suspensions, which in turn can have an influence on efficiency or selectivity of mineral separation processes. This can for instance include tracking of particle motion, study of the microstructure of the magnetic particle matrix and capturing mechanisms of non-magnetic particles.

A comparison between the approach discussed in this paper and work by other groups is made in Table 1.

<table>
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<th>Murairu 2013</th>
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<td>Stokes drag</td>
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<td>Laminar or primitive turbulent</td>
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4.1 Further work

In the current work, an approach with either completely magnetic or non-magnetic particles was implemented and tested. The method is thus currently not adapted for studying mixed grain type particles. Further work could include the development of interaction models where magnetic susceptibilities could be varied, for instance based on particle size. A long term possibility could be to also implement susceptibility distributions for different particle size classes or by assignment of magnetic susceptibilities to individual particles by random procedures.

So far, the work has been qualitatively compared with flow cell experiments (Stener et al. 2014). Systematic comparison or validation type tests could include more detailed comparison of particle solids concentration or structure near the location of the external magnet. This would allow evaluation of the implemented model and possibly also adjustment of models or model parameters in order to produce a more accurate behaviour. Another possibility could be to quantitatively study non-magnetic particle capturing within the matrix of magnetic particles.
This would however require a possibility to extract the magnetic cake from the flow cell and analyse the fraction of non-magnetic particles within.

5 References


